CSS 322 – QUIZ 5 ANSWERS

First name: _____ Last name: _____

ID: _____ Total Marks: _____
out of 5

Question 1 [2 marks]

Perform the following calculations using modular arithmetic:

a) Modular 13: 12 x 6
b) Modular 17: 8 – 15
c) Modular 15: 12 ÷ 6
d) Modular 17: 12 ÷ 6

Answers

a. 12 x 6 = 72. 72 mod 13 = 7

b. 8 + additive_inverse(15) = 8 + 2 = 10

c. 6 does not have a multiplicative inverse in mod 15, and no answer.

d. 12 x mult_inverse(6) = 12 x 3 = 36. 36 mod 17 = 2

Question 2 [2 marks]

Calculate the following:

a) Ø(16)
b) Ø(17)
c) Ø(13)
d) Ø(221)

Answers

a. Factors of 16 are 2, 4, 8, 16. Numbers relatively prime to 16 are: 1, 3, 5, 7, 9, 11, 13, 15. Hence answer is 8.

b. 17 is prime, hence answer is 16.

c. 13 is prime, hence answer is 12.

d. 221 = 17 x 13, hence answer is 16 x 12 = 192.

Question 3 [1 mark]

Euler's theorem states that, for two relatively prime numbers, *a* and *n*:

$$a^{\phi(n)} \equiv 1 (\mathrm{mod}\, n)$$

Derive the answer of: 15^8 mod 16? You must show (or explain) calculations/derivation.

Answer

Since Ø(16) = 8, and 15 is relatively prime to 16, then Eulers theorem applies if a = 15 and n = 16. Therefore 15^8 mod 16 = $a^{Ø(n)}$ mod 16 = 1