Pseudo-Random Numbers and Stream Ciphers

CSS441: Security and Cryptography

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Contents

Random Numbers

Use of Random Numbers

- Key distribution and authentication schemes
- Generation of session keys or keys for RSA
- Generation of bit stream for stream ciphers

Randomness

- Uniform distribution: frequency of occurrence of 1's and 0's approximately equal
- Independence: no sub-sequence can be inferred from others

Unpredictability

- Hard to predict next value in sequence

TRNG, PRNG and PRF

True Random Number Generator

- Non-deterministic source, physical environment
- Detect ionizing radiation events, leaky capacitors, thermal noise from resistors or audio inputs
- Mouse/keyboard activity, I/O operations, interrupts
- Inconvenient, small number of values

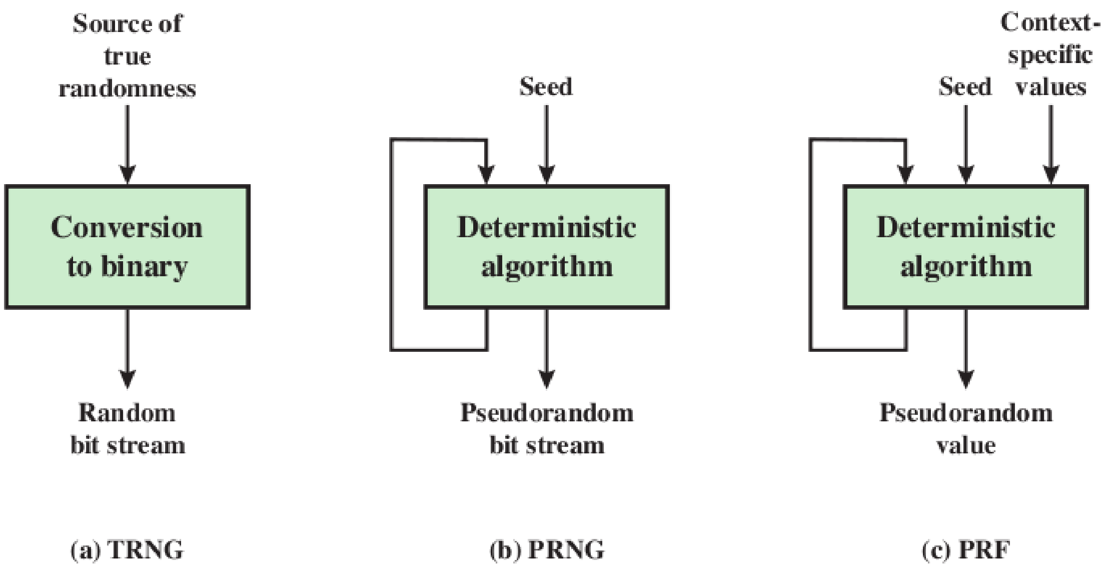
Pseudo Random Number Generator

- Deterministic algorithms to calculate numbers in "relatively random" sequence
- Seed is algorithm input
- Produces continuous stream of random bits

Pseudo Random Function

- Same as PRNG but produces string of bits of some

Random and Pseudo-Random Number Generators

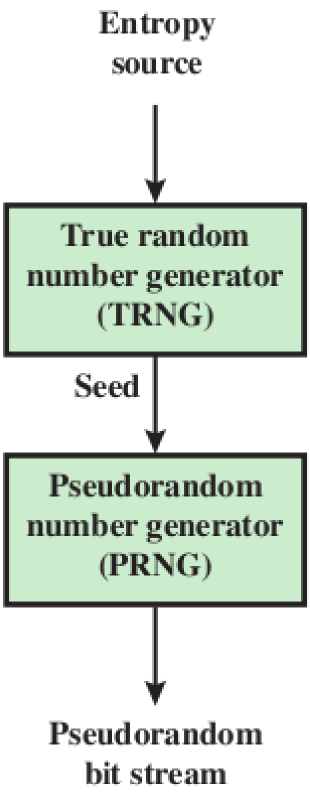


Requirements of PRNG

Hard to determine pseudo-random stream if don't know seed (but know algorithm)

- Randomness
  - Test for uniformity, scalability, consistency
  - Examples: Frequency, runs, compressability
- Unpredictability
  - Forward and backward unpredictability
- Seed must be secure
  - Use TRNG to generate seed

Generation of Seed Input to PRNG



Contents

Linear Congruential Generator

Parameters:

- m, the modulus, $m > 0$
- a, the multiplier, $0 < a < m$
- c, the increment, $0 \le c < m$
- X_0, the seed, $0 \le X_0 < m$

Generate sequence of pseudo-random numbers, $\{X_n\}$:

$$X_{n+1} = (aX_n + c) \bmod m$$

Choice of a, c and m is important:

- m should be large, prime, e.g. $2^{31} - 1$
- If c=0, few good values of a, e.g. $7^5 = 16807$

If attacker knows parameters and one number, can easily determine subsequent numbers

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RC4

Blum Blum Shub Generator

Parameters:

- p, q: large prime numbers such that $p \equiv q \equiv 3 \pmod 4$
- $n = p \times q$
- s, random number relatively prime to n

Generate sequence of bits, B_i:

$$\begin{aligned} X_0 &= s^2 \bmod n \\ \text{for } i &= 1 \to \infty \\ X_i &= (X_{i-1})^2 \bmod n \\ B_i &= X_i \bmod 2 \end{aligned}$$

Cryptographically secure pseudo-random bit generator

Example Operation of BBS Generator

$n = 192649 = 383 \times 503,\ s = 101355$

i	X_i	B_i
0	20749	
1	143135	1
2	177671	1
3	97048	0
4	89992	0
5	174051	1
6	80649	1
7	45663	1
8	69442	0
9	186894	0
10	177046	0

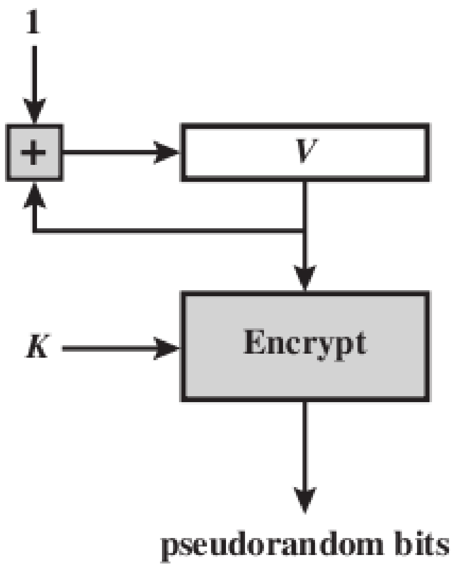
i	X_i	B_i
11	137922	0
12	123175	1
13	8630	0
14	114386	0
15	14863	1
16	133015	1
17	106065	1
18	45870	0
19	137171	1
20	48060	0

Contents

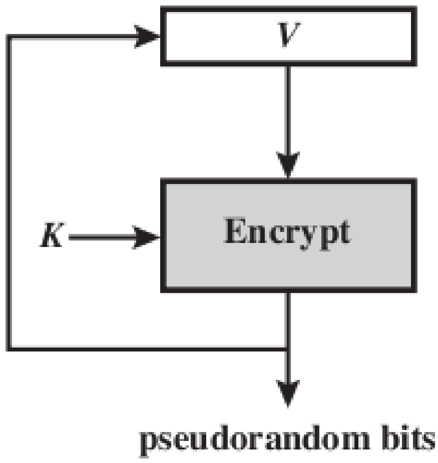
PRNG Mechanisms Based on Block Ciphers

Use symmetric block ciphers (e.g. AES, DES) to produce pseudo-random bits

- Seed is encryption key, K, and value V (which is updated)



Counter Mode



OFB Mode

ANSI X9.17 PRNG

Cryptographically secure PRNG using Triple DES

Parameters:

- 64-bit date/time representation, DT_i
- 64-bit seed value, V_i
- Pair of 56-bit DES keys, K_1 and K_2

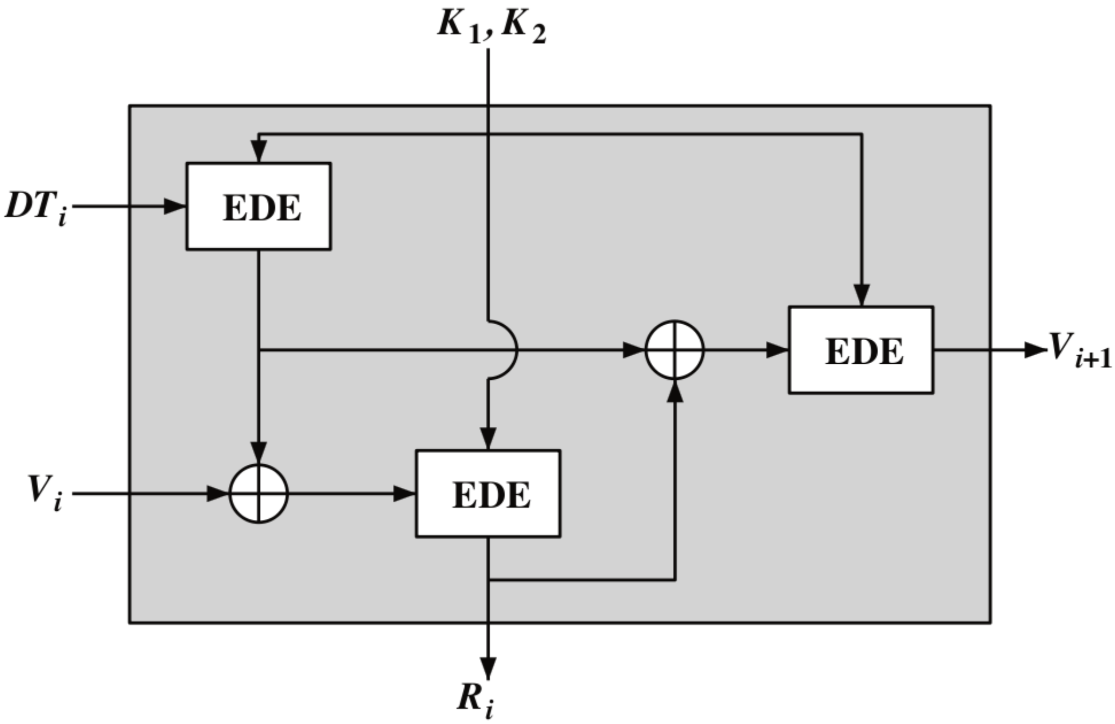
Operation:

- Uses Triple DES three times
- (see next slide)

Output:

- 64-bit pseudo-random number, R_i
- 64-bit seed value, V_{i+1}

ANSI X9.17 PRNG



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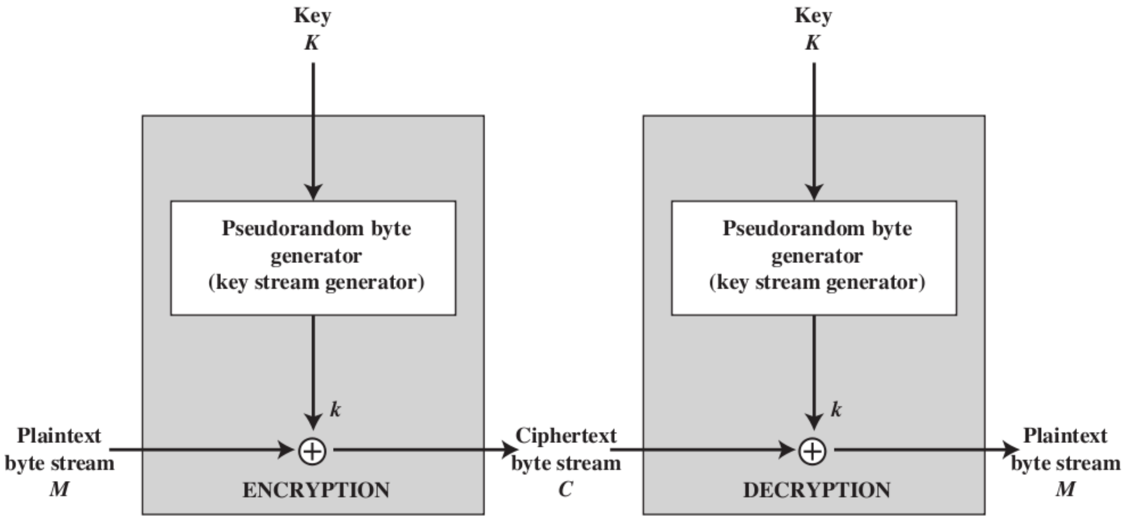
Stream Ciphers

Encrypt one byte at a time by XOR with pseudo-random byte



Output of generator is called keystream

Design Criteria for Stream Ciphers

Important Considerations

- Encryption sequence should have large period
- Keystream should approximate true random number stream
- Key must withstand brute force attacks

Comparison to Block Ciphers

- Stream ciphers often simpler to implement, faster
- Block ciphers can re-use keys

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RC4

- Designed by Ron Rivest in 1987
- Used in secure web browsing and wireless LANs
- Very simple and efficient implementation
- Can use variable size key: 8 to 2048 bits
- Several theoretical limitations of RC4
  - No known attacks if use 128-bit key and discard initial values of stream
  - RC4 is used in WEP (shown to be weak security for wireless LANs)—problem with how keys are used, not RC4 algorithm

RC4 Algorithm

Parameters and Variables

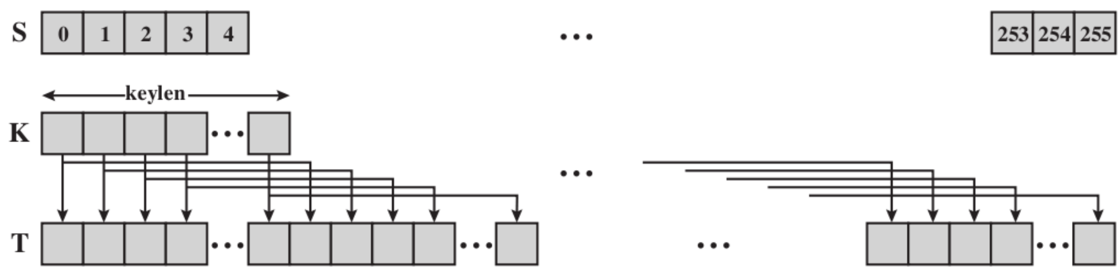
- Variable length key, K, from 1 to 256 Bytes
- State vector, S, 256 Bytes
- Temporary vector, T, 256 Bytes
- A byte from keystream, k, generated from S

Steps

1. Initialise S to values 0 to 255; initialise T with repeating values of key, K
2. Use T to create initial permutation of S
3. Permutate S and generate keystream, k from S
4. Encrypt a byte of plaintext, p, by XOR with k

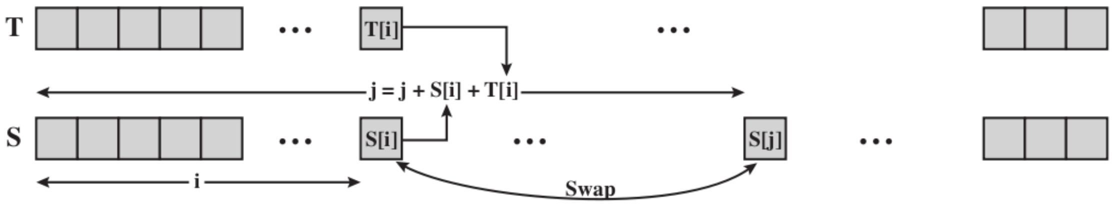
Initial State of S and T

```
for i = 0 to 255 do
   S[i] = i;
   T[i] = K[i mod keylen];
```



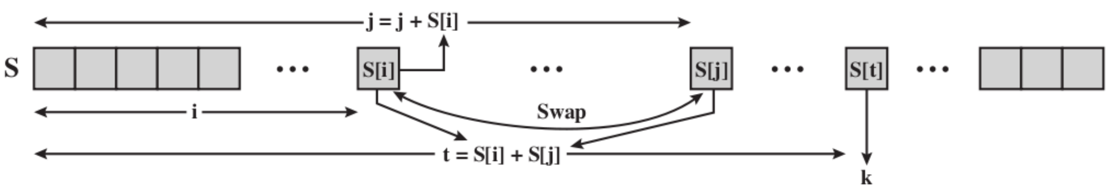
Initial Permutation of S

```
j = 0;
for i = 0 to 255 do
   j = (j + S[i] + T[i]) mod 256;
Swap (S[i], S[j]);
```



Stream Generation

```
i, j = 0;
while (true)
   i = (i + 1) mod 256;
   j = (J + S[i]) mod 256;
   Swap (S[i], S[j]);
   t = (S[i] + S[j]) mod 256;
   k = S[t];
```



To encrypt: $C = p \text{ XOR } k$
To decrypt: $p = C \text{ XOR } k$